

Introduction to Financial Economics Exercise 2

1. (36 points) Determine if the following functionals are positive or strictly positive.
 - (a) (6 points) $F(x_1, x_2) = x_1^2 + x_2^3$;
 - (b) (6 points) $F(x_1, x_2, x_3) = x_1^2 + x_2^2 + x_3^2$;
 - (c) (6 points) $F(x_1, x_2, x_3) = x_1^2 + x_2^2 + x_3$;
 - (d) (6 points) $F(x_1, x_2, x_3) = 2x_1 + x_2 + 4x_3$;
 - (e) (6 points) $F(x_1, x_2, x_3) = 2x_1 - x_2 + 4x_3$;
 - (f) (6 points) $F(x_1, x_2, x_3) = 2x_1 + x_3^2$.
2. (70 points) Find $q_u(Z)$ and $q_l(Z)$ and one valuation functional $Q(Z)$ such that the model is arbitrage-free in the following cases.
 - (a) (14 points) Two states and one security with $P_1 = 1$, $X_1 = (2, 2)$. The contingent claim $Z = (2, 1)$.
 - (b) (14 points) Two states and one security with $P_1 = 1$, $X_1 = (1, 3)$. The contingent claim $Z = (2, 1)$.
 - (c) (14 points) Three states and one security with $P_1 = 1$, $X_1 = (1, 2, 3)$. The contingent claim $Z = (2, 1, 1)$.
 - (d) (14 points) Three states and two securities with $P_1 = 1$, $X_1 = (2, 2, 2)$, $P_2 = 1$, $X_2 = (1, 2, 3)$. The contingent claim $Z = (3, 2, 1)$.
 - (e) (14 points) Three states and two securities with $P_1 = 1$, $X_1 = (2, 2, 2)$, $P_2 = 1$, $X_2 = (3, 1, 4)$. The contingent claim $Z = (1, 2, 3)$.
3. (28 points) Suppose that there are three states and two securities with

$$P = (P_1, P_2)^T = (1, 1)^T, \quad X_1 = (2, 2, 2), \quad X_2 = (1, 2, 3).$$

- (a) (8 points) Consider $Z_1 = (4, 2, 1)$, find $q_u(Z_1)$ and $q_l(Z_1)$.
- (b) (6 points) Find a strictly positive valuation functional.

(c) (8 points) Consider $Z_2 = (1, 3, 3)$, find $q_u(Z_2)$ and $q_l(Z_2)$.

(d) (6 points) Discuss the relation of the valuation functional constructed in (b) and the functionals $q_u(Z_2)$ and $q_l(Z_2)$ in (c).

Due to November 10. Total: 134 points.