

## Introduction to Financial Economics      Exercise 2

1. (36 points) Determine if the following functionals are positive or strictly positive.

- (a) (6 points)  $F(x_1, x_2) = x_1^2 + x_2^3$ ;
- (b) (6 points)  $F(x_1, x_2, x_3) = x_1^2 + x_2^2 + x_3^2$ ;
- (c) (6 points)  $F(x_1, x_2, x_3) = x_1^2 + x_2^2 + x_3$ ;
- (d) (6 points)  $F(x_1, x_2, x_3) = 2x_1 + x_2 + 4x_3$ ;
- (e) (6 points)  $F(x_1, x_2, x_3) = 2x_1 - x_2 + 4x_3$ ;
- (f) (6 points)  $F(x_1, x_2, x_3) = 2x_1 + x_3^2$ .

2. (70 points) Find  $q_u(Z)$  and  $q_l(Z)$  and one valuation functional  $Q(Z)$  such that the model is arbitrage-free in the following cases.

- (a) (14 points) Two states and one security with  $P_1 = 1$ ,  $X_1 = (2, 2)$ . The contingent claim  $Z = (2, 1)$ .
- (b) (14 points) Two states and one security with  $P_1 = 1$ ,  $X_1 = (1, 3)$ . The contingent claim  $Z = (2, 1)$ .
- (c) (14 points) Three states and one security with  $P_1 = 1$ ,  $X_1 = (1, 2, 3)$ . The contingent claim  $Z = (2, 1, 1)$ .
- (d) (14 points) Three states and two securities with  $P_1 = 1$ ,  $X_1 = (2, 2, 2)$ ,  $P_2 = 1$ ,  $X_2 = (1, 2, 3)$ . The contingent claim  $Z = (3, 2, 1)$ .
- (e) (14 points) Three states and two securities with  $P_1 = 1$ ,  $X_1 = (2, 2, 2)$ ,  $P_2 = 1$ ,  $X_2 = (3, 1, 4)$ . The contingent claim  $Z = (1, 2, 3)$ .

3. (28 points) Suppose that there are three states and two securities with

$$P = (P_1, P_2)^T = (1, 1)^T, \quad X_1 = (2, 2, 2), \quad X_2 = (1, 2, 3).$$

- (a) (8 points) Consider  $Z_1 = (4, 2, 1)$ , find  $q_u(Z_1)$  and  $q_l(Z_1)$ .
- (b) (6 points) Find a strictly positive valuation functional.

(c) (8 points) Consider  $Z_2 = (1, 3, 3)$ , find  $q_u(Z_2)$  and  $q_l(Z_2)$ .

(d) (6 points) Discuss the relation of the valuation functional constructed in (b) and the functionals  $q_u(Z_2)$  and  $q_l(Z_2)$  in (c).

Due to November 10. Total: 134 points.