Introduction to Financial Economics   Exercise 1

1. (36 points) Find the payoff matrices, asset span and security returns of the following market models. Furthermore, determine if the market is complete.

   (a) (12 points) $P = (P_1, P_2)^T = (1, 1)^T$; $X_1 = (1.2, 1.2, 1.2)$, $X_2 = (1, 1.2, 2)$.

   (b) (12 points) $P = (P_1, P_2, P_3)^T = (1, 1, 1)^T$; $X_1 = (1.2, 1.2, 1.2)$, $X_2 = (0.8, 1, 1.2)$, $X_3 = (0.9, 1.1, 1.3)$.

   (c) (12 points) $P = (P_1, P_2, P_3, P_4)^T = (1, 1, 1, 1)^T$; $X_1 = (1, 1, 1)$, $X_2 = (0.8, 1.2, 1.2)$, $X_3 = (0.8, 0.8, 1.2)$, $X_4 = (0.5, 2, 4)$.

2. (32 points) Determine if there exists an arbitrage opportunity in the following models. If yes, find an arbitrage.

   (a) (8 points) $P = (P_1, P_2)^T = (1, 1)^T$; $X_1 = (1, 1)$, $X_2 = (0.8, 2)$.

   (b) (8 points) $P = (P_1, P_2)^T = (1, 1)^T$; $X_1 = (1.2, 1.2, 1.2)$, $X_2 = (1.1, 2, 2)$.

   (c) (8 points) $P = (P_1, P_2)^T = (1, 1)^T$; $X_1 = (1.2, 1.2, 1.2)$, $X_2 = (1.2, 2, 2)$.

   (d) (8 points) $P = (P_1, P_2)^T = (1, 1)^T$; $X_1 = (1, -1, 1)$, $X_2 = (-1, 1, 1)$.

3. (24 points) Determine if the law of one price holds. If yes, find the corresponding payoff pricing functional.

   (a) (12 points) $P = (P_1, P_2)^T = (1, 1)^T$; $X_1 = (1.2, 1.2, 1.2)$, $X_2 = (0.8, 1, 1.2)$.

   (b) (12 points) $P = (P_1, P_2, P_3)^T = (1, 1, 1)^T$; $X_1 = (1, 1, 1)$, $X_2 = (0.8, 1, 1.2)$, $X_3 = (1.2, 1, 0.8)$.

4. (24 points) Find the payoff pricing functionals in the following cases. Using these functionals to check if the models are arbitrage-free or strong arbitrage-free.

   (a) (12 points) $P = (P_1, P_2)^T = (1, 1)^T$; $X_1 = (2, 2, 2)$, $X_2 = (3, 4, 5)$.

   (b) (12 points) $P = (P_1, P_2, P_3)^T = (1, 1, 1)^T$; $X_1 = (1, 1, 1)$, $X_2 = (1, 2, 2)$, $X_3 = (2, 1, 2)$.

Due to October 20. Total: 116 points.